Modified Bianchi type- I cosmological model by using cosmological constant

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Abstract

We investigate the modified Bianchi type I cosmological model by using cosmological constant i.e. Cosmological constant is proportional to the square of the Hubble parameter. In this condition Einstein's field equations with variable cosmological constant is considered in presence of perfect fluid.

Keywords: Bianchi type I, universe, varying cosmological constant ∧, cosmology, Hubble parameter.

MSC 2010 Classification: 83C05, 83C15.

1. Introduction

Cosmology is study of large-scale structures of universe. Very important discovery of recent times is the cosmological constant problem, which is very interesting to all researchers. In Einstein's field equation cosmological constant ∧ present. Cosmological models with variable G and ∧ have been studied by a number of researchers$[1-7]$ for a humogen and isotropic FRW line element. Also, Bianchi type -I models are studied by using variable G and ∧. $[8-14]$. Tiwari and Sonia investigated the Bianchi type -I string cosmological model with bulk viscosity and time dependent ∧ term. LRS Bianchi type -I cosmological model with polytropic equation of state R. K, Dubey and Shishir. In this paper we investigate the modified homogeneous anisotropic Bianchi type -I space time and variable cosmological constant. Our calculation matter in form of a perfect fluid by using cosmological constant is proportional to the square of Hubble parameter.

Metric and field equations-

We consider line element for spatially homogeneous and anisotropic Bianchi type I space time described.

$$ds^2 = -dt^2 + L^2dx_1^2 + M^2dx_2^2 + P^2 dx_3^2 \qquad (1)$$

Where L, M, P are functions of t only.

Let us assume that cosmic matter is represented by the energy momentum tensor of a perfect fluid.

$$T_{ls} = (\rho + p)\, v\, v_s + p\, g_{ls} \qquad (2)$$

Where ρ and p are present energy density and pressure.

The four-velocity vector of particles and x^l is the unit space like vector represents the direction of the fluid satisfying the relation

$$v_l v^l = -1 \text{ and} \qquad (3)$$

$$v_l v^l = 0$$

in a co-moving system, we get

$$v^l = (0, 0, 0, 1), \qquad x^l = (\tfrac{1}{L}, 0, 0, 0) \qquad (4)$$

We take equation of state

$p = \eta\rho$, $0 \le \rho \le 1$ (5)

v^l is four velocity vectors of the fluid satisfying

$g_{ls} v^l v^s = -1$ (6)

The Einstein's equations with varying $\wedge$ in suitable

$R_{ls} - \frac{1}{2} R g_{ls} = - T_{ls} + \wedge g_{ls}$ (7)

Let spatial volume V an average scale factor of the model (1) defined as $V = \varepsilon = (LMP)^{\frac{1}{3}}$ (8)

Hubble parameter H is defined as

$H = \frac{\dot{\varepsilon}}{\varepsilon} = \frac{1}{3}\left(\frac{\dot{L}}{L} + \frac{\dot{M}}{M} + \frac{\dot{P}}{P}\right)$ (9)

Here ' . ' denotes ordinary time derivative.

$H = \frac{1}{3}(H_x + H_y + H_z)$

Here $H_x = \frac{\dot{L}}{L}$, $H_y = \frac{\dot{M}}{M}$ and $H_Z = \frac{\dot{P}}{P}$

H_x, H_Y, H_z are Hubble factors.

From metric (1) and energy moment tensor (2) in the co-moving system of the coordinates

The Field equation (4) fields

$\frac{\ddot{M}}{M} + \frac{\ddot{P}}{P} + \frac{\dot{M}\dot{P}}{MP} = -p + \wedge$ (10)

$\frac{\ddot{L}}{L} + \frac{\ddot{P}}{P} + \frac{\dot{L}\dot{P}}{LP} = -p + \wedge$ (11)

$\frac{\ddot{L}}{L} + \frac{\ddot{M}}{M} + \frac{\dot{L}\dot{M}}{LM} = -p + \wedge$ (12)

$\frac{\dot{L}\dot{M}}{LM} + \frac{\dot{M}\dot{P}}{MP} + \frac{\dot{L}\dot{P}}{LP} = \rho + \wedge$ (13)

Our view the vanishing divergence of Einstein tensor

$8\pi G\left[\dot{\rho} + (\rho + p)\left(\frac{\dot{L}}{L} + \frac{\dot{M}}{M} + \frac{\dot{P}}{P}\right)\right] + \dot{\wedge} + 8\pi\dot{G}$ (14)

If $\dot{\rho} + (\sigma + p)\left(\frac{\dot{L}}{L} + \frac{\dot{M}}{M} + \frac{\dot{P}}{P}\right) = 0$ (15)

From equation (14)

$8\pi\rho\dot{G} + \dot{\wedge} = 0$ (16)

Employing that cosmological constant $\wedge$ and G is gravitational constant.

$\rho = \frac{K_1}{\varepsilon^2}$ (17)

θ expansion scalar, σ shear scalar and q deceleration parameter are given by $\theta = 3H = 3\frac{\dot{\varepsilon}}{\varepsilon}$

The Einstein's field equation (10)-(13) in terms of Hubble parameter H, shear parameter σ and deceleration parameter q can be written as

$$H^2 (2q-1) - \sigma^2 = \rho - \wedge \tag{18}$$

$$3H^2 - \sigma^2 = \rho + \eta \tag{19}$$

$$Q = -1 - \frac{\dot{H}}{H^2} = \frac{\varepsilon\ddot{\varepsilon}}{\dot{\varepsilon}^2} \tag{20}$$

From equation (10), (11) and (12)

$$\frac{\dot{L}}{L} - \frac{\dot{M}}{M} = \frac{l_1}{\varepsilon^3} \tag{21}$$

$$\frac{\dot{L}}{L} - \frac{\dot{P}}{P} = \frac{l_2}{\varepsilon^3} \tag{22}$$

$$\frac{\dot{M}}{M} - \frac{\dot{P}}{P} = \frac{l_3}{\varepsilon^3} \tag{23}$$

Where l_1, l_2 and l_3 are constant of integration

Again, integrating equations (21) – (23) we get

$$\frac{L}{M} = n_1 \exp (l_1 \int \frac{1}{\varepsilon^3} dt) \tag{24}$$

$$\frac{L}{P} = n_2 \exp (l_2 \int \frac{1}{\varepsilon^3} dt) \tag{25}$$

$$\frac{M}{P} = n_3 \exp (l_3 \int \frac{1}{\varepsilon^3} dt) \tag{26}$$

Where n_1, n_2 and n_3 are integration constant

From equation (20)

$$\frac{3\sigma^2}{\theta^2} = 1 - \frac{3\rho}{\theta^2} - \frac{3\wedge}{\theta^2} \tag{27}$$

$$\wedge \geq 0,\ 0 < \frac{\sigma^2}{\theta^2} \leq \frac{1}{3},\ 0 < \frac{\rho}{\theta^2} < \frac{1}{3}$$

$$\wedge \geq 0,\ 0 < \frac{\rho}{\theta} \leq \frac{1}{\sqrt{3}},\ 0 < \frac{\rho}{\theta^2} < \frac{1}{3}$$

Equation (27) can be written as

$$\frac{3\sigma^2}{3H^2} = 1 - \frac{3\rho}{3H^2} - \frac{\wedge}{3H^2} = 1 - \frac{\rho}{\rho_c} - \frac{\rho_v}{\rho_c} \tag{28}$$

Where $\rho_c = 3H^2$ is critical density and $\rho_v = \wedge$ is vacuum density

$$\dot{\theta} = \frac{-3}{2}(\rho + p) - 3\sigma^2 \tag{29}$$

According to condition $\wedge \propto H^2$

$$\wedge = \nu H^2 \tag{30}$$

Where ν is the positive constant.

Let vacuum matter density

$\mu = \frac{\wedge}{\rho}$

From equation (19) and (28), we get $\nu = \frac{3\mu}{1+\mu}(1 - \frac{\sigma^2}{27\theta^2})$ (31)

For stiff fluid, equation (18), (19) and (30) we get new relation

$\dot{H} + (3-\mu) H^2 = 0$ (32)

$\frac{d}{dt}(H) + (3-\mu) H^2 = 0$ (33)

Integrating, we have

$\varepsilon = [(3-\mu)(v_1 t + v_{2)}]^{\frac{1}{3-\mu}}$ (34)

And geometrical parameter can be easily obtained the matter density ρ , pressure p, cosmological constant $\wedge$,shear scalar σ and expansion scalar θ are given by

$\rho = p = [n_4(3-\mu)(v_1 t + v_{2)}]^{\frac{-6}{3-\mu}}$

$\wedge = \mu v_1^2 \, [(3-\mu)(v_1 t + v_{2)}]^{-2}$

$\theta = \frac{H}{3} = \frac{c_1}{3}[(3-\mu)(v_1 t + v_{2)}]^{-1}$

$\sigma = (v_1 t + v_2)^{\frac{-3v_1^2}{3-\mu}} . v_3$

$\nu = \frac{\wedge}{\rho} = \frac{\mu v_1^2}{n_4}\{(3-\mu)(v_1 t + v_{2)}\}^{\frac{3\mu}{3-\mu}}$

The deceleration parameter q for this model is q = 2 - μ

$\upsilon = \varepsilon^3 = \{(3-\mu)(v_1 t + v_{2)}\}^{\frac{3}{3-\mu}}$

2. Conclusion:

In this paper t= $\frac{-v_2}{v_1}$, the spatial volume V=0 if t= $\frac{-v_2}{v_1}$, then expansion scaler θ is infinite.

Where $v_1 and v_2$ are the constant of integration.

$H = \frac{\dot{\varepsilon}}{\varepsilon} = v_1 \, [(3-\mu)(v_1 t + v_{2)}]^{-1}$ (35)

$L = \{(3-\mu)(v_1 t + v_{2)}\}^{\frac{1}{3-\mu}} . \, e^{\frac{2l_1 + l_2}{6(3-\mu)(v_1 t + v_2)}}$

$M = \{(3-\mu)(v_1 t + v_{2)}\}^{\frac{1}{3-\mu}} . \, e^{\frac{\langle_2 - l_1}{3\{(3-\mu)(v_1 t + v_{2)}\}^{\frac{3}{3-\mu}}}}$

$P = \{(3-\mu)(v_1 t + v_{2)}\}^{\frac{1}{3-\mu}} . \, e^{\frac{(2l_2 - l_1)}{2\{(3-\beta)(v_1 t + v_{2)}\}^{\frac{3}{3-\mu}}}}$

We put the value L, M, P in metric (1) we get

$$ds^2 = -dt^2 + \{(3-\beta)(v_1 t + v_2)\}^{\frac{2}{3-\mu}} + e^{\frac{2l_1+l_2}{3\{(3-\beta)(v_1t+v_2\}^{\frac{3}{3-\mu}}}} dx_1^2 + e^{\frac{2(l_2-l_1)}{3\{(3-\beta)(v_1t+v_2\}^{\frac{3}{3-\mu}}}} dx_2^2 + e^{\frac{2l_2-l_1}{\{(3-\beta)(v_1t+v_2\}^{\frac{3}{3-\mu}}}} dx_3^2$$

3. Some physical and geometrical properties:

For the model equation (23), the physics involving with zero volume with infinite rate of expansion.

H t = - $\frac{v_2}{v_1}$ then average scale factor R = 0 which shows that during initial age of space time exhibits a point type singularity.

At t = - $\frac{v_2}{v_1}$, $\rho \to \infty$, $\sigma \to \infty$, we see that time increases the average scale factor ε and spatial volume increases but expansion scalar decreases i.e. the rate of expansion slows down. When $t \to \infty$, then $R \to \infty$, $\gamma \to \infty$, $\wedge \to \infty$,then the universe model represents an empty universe for $t \to \infty$. This result is in agreement with observations obtained by many astronomers.

Our calculation represents the Bianchi type I cosmological model containing a stiff fluid with cosmological term $\wedge = \gamma H^2$. It is found that deceleration parameter q for the model is 2 at $\mu = 0$. The cosmological constant $\wedge$, being very large at the initial stage later relaxes to a genuine cosmological constant according to recent observations. Overall conclusion model asymptomically tends to the de-sitter universe.

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